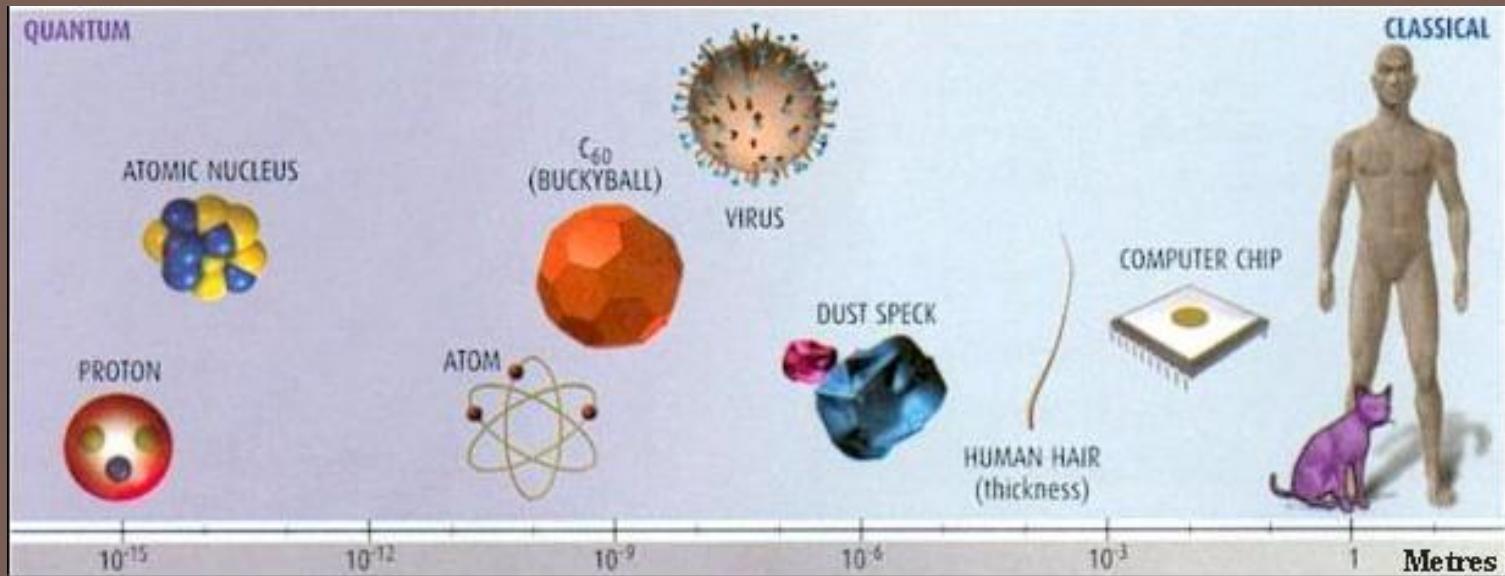


REDRAWING THE QUANTUM-CLASSICAL BOUNDARY



Mishkat Bhattacharya

TALK OUTLINE

- Introduction
- Reaching the macroscopic ground state
- The quantum limit of macroscopic position measurement
- Conclusion

AMO THEORY GROUP



Rochester Institute of Technology

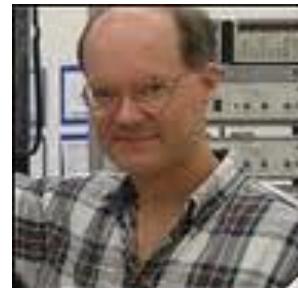
- UG
- N. Cawley
 - M. Eggleston
 - T. Godat
 - S. Igbokwe
 - Z. Howard
 - E. Munro
 - M. Schumacher
- G
- S. Preble
 - E. Hach
- F

Hao Shi*



*2013
LeRoy Apker Award
of the APS

J. Lawall



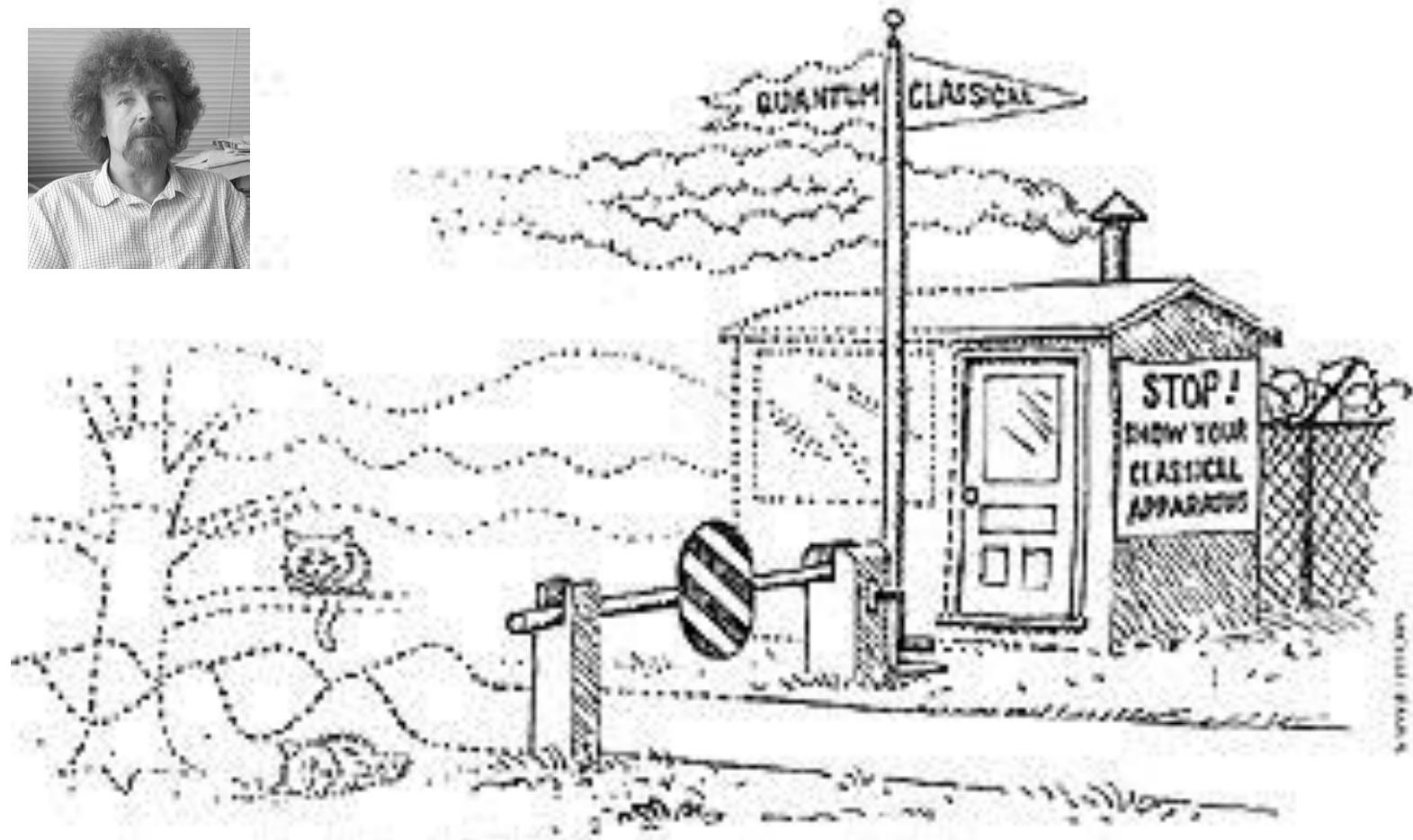
NIST
Gaithersburg

M. Kleinert
(Willamette
College)

Kevin Wright
(Dartmouth)

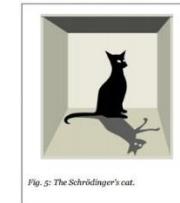
Miguel Alonso
Nick Vamivakas
(U Rochester)

LIFE AT THE QUANTUM-CLASSICAL BOUNDARY



* W. Zurek, Los Alamos

QUANTUM MECHANICS AT THE MACROSCALE



Pig. 5: The Schrödinger's cat.

1. Fundamental interest: Bottom up

- QM has been verified at the level of molecules, atoms, nucleons, quarks....
- Nonclassical phenomena like superposition, tunneling, entanglement...
- What happens at larger scales ? Does Schrodinger's equation break down ?

G. C. Ghirardi, A. Rimini, and T. Weber, PRD **34**, 470 (1986)

A. Bassi and G. C. Ghirardi, Phys. Rep. **379**, 257 (2003)

S. L. Adler and A. Bassi, Science **325**, 275 (2009)

O. Romero-Isart, PRA **84**, 052121 (2011)

2. Applied interest: Top down

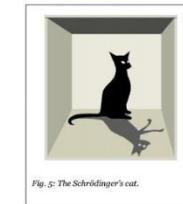
- What are the measurement limits to a human-made sensor ?
- Gravitational wave detection, atomic force microscopy

McClelland, et. al, *Laser & Photonics Reviews*, 5(5), 677(2011).

M. Poggio et. al, PRL **99**, 017201(2007)

LaHaye et. al, *Nature* **459**, 960 (2009).

QUANTUM MECHANICS AT THE MACROSCALE



Pig. 5: The Schrödinger's cat.

Putting Mechanics into Quantum Mechanics

Nanoelectromechanical structures are starting to approach the ultimate quantum mechanical limits for detecting and exciting motion at the nanoscale. Nonclassical states of a mechanical resonator are also on the horizon.

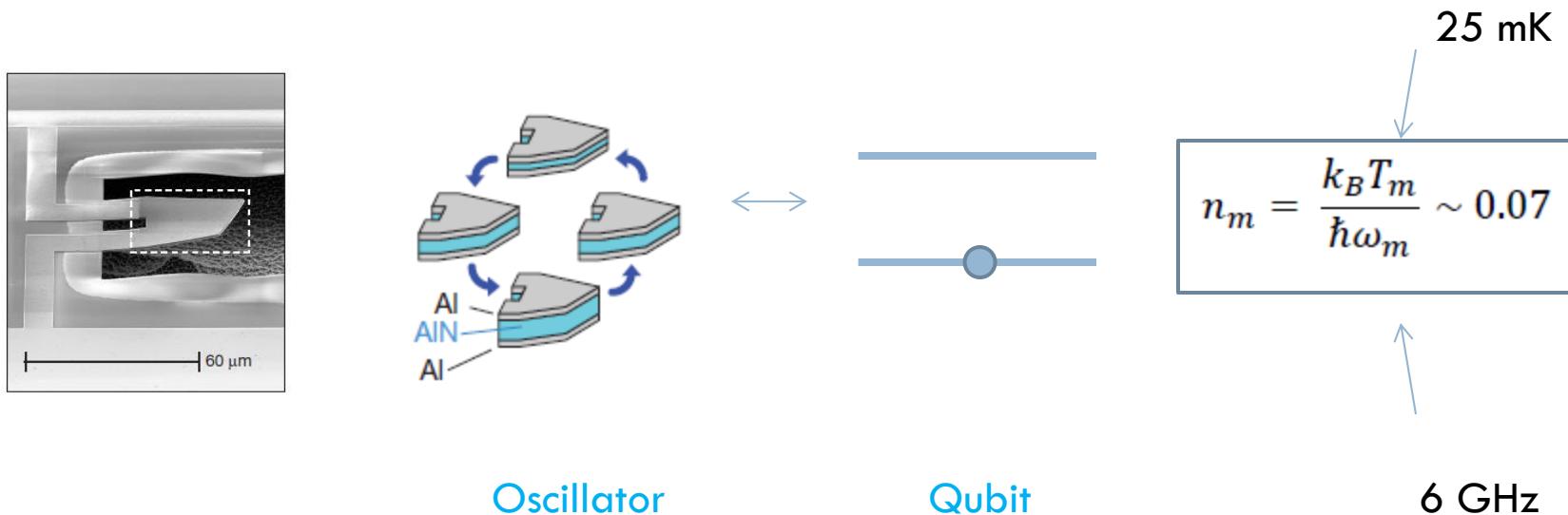
Keith C. Schwab and Michael L. Roukes

- Physics Today, July 2005

CRYOGENIC COOLING OF A MECHANICAL OSCILLATOR

Quantum ground state and single-phonon control of a mechanical resonator

A. D. O'Connell¹, M. Hofheinz¹, M. Ansmann¹, Radoslaw C. Bialczak¹, M. Lenander¹, Erik Lucero¹, M. Neeley¹, D. Sank¹, H. Wang¹, M. Weides¹, J. Wenner¹, John M. Martinis¹ & A. N. Cleland¹



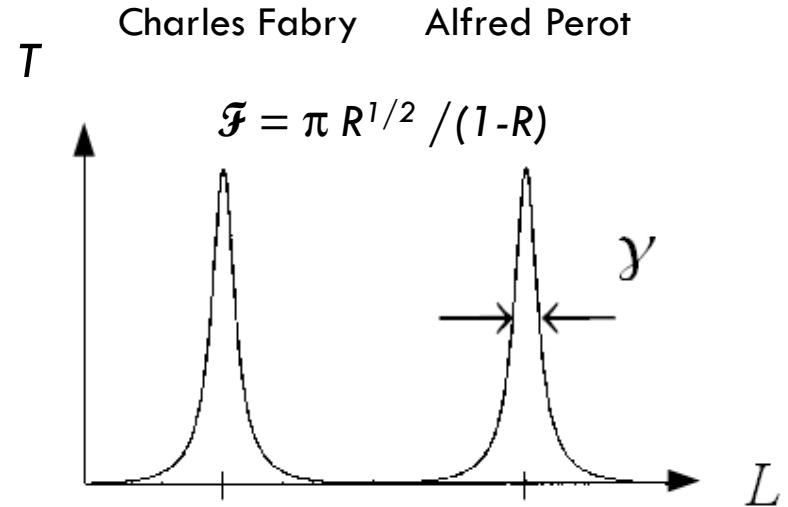
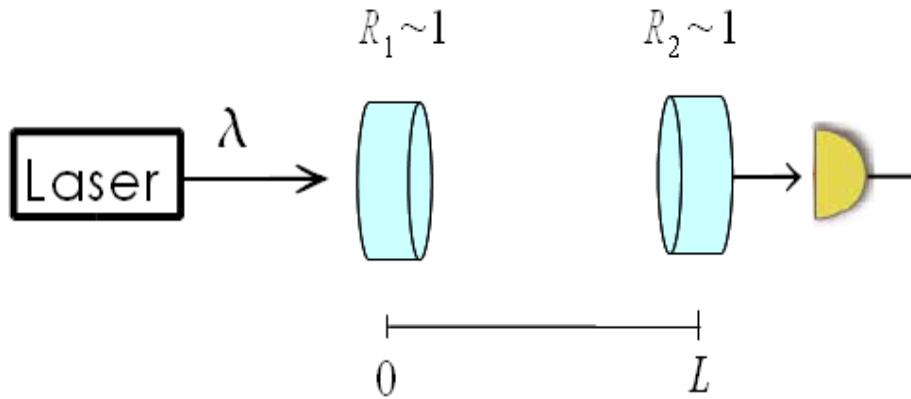
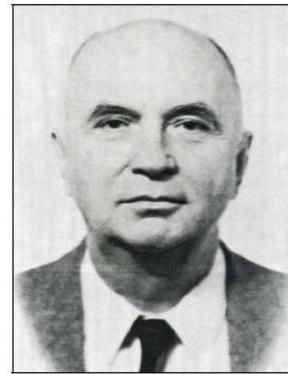
* 2010 Science magazine Breakthrough of the Year

OPTOMECHANICAL COOLING

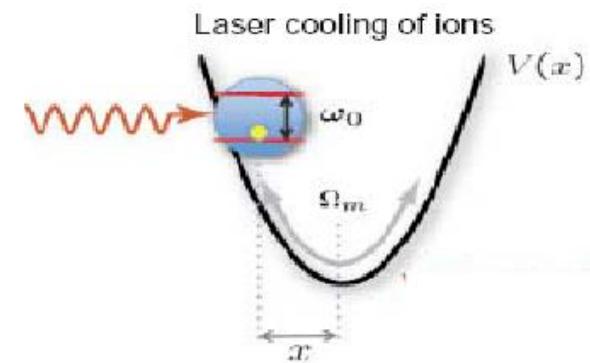
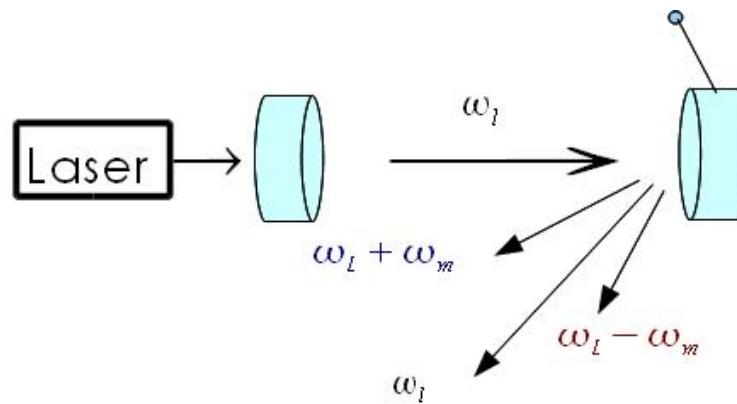
□ The Fabry-Perot cavity (1897)



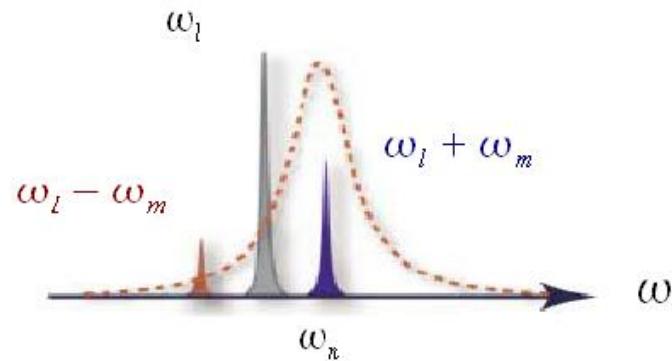
* V. Braginsky



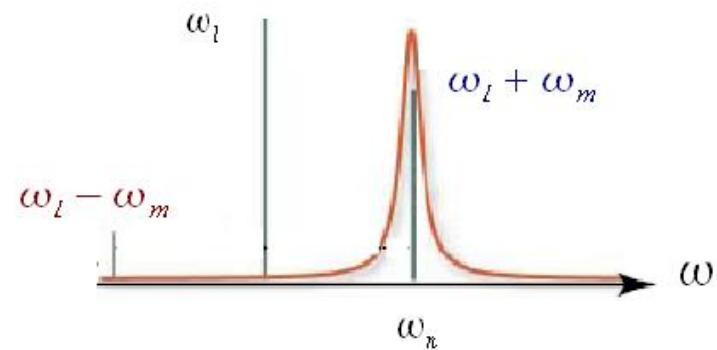
RESONATOR-BASED COOLING



Doppler cooling : $\omega_m < \gamma$



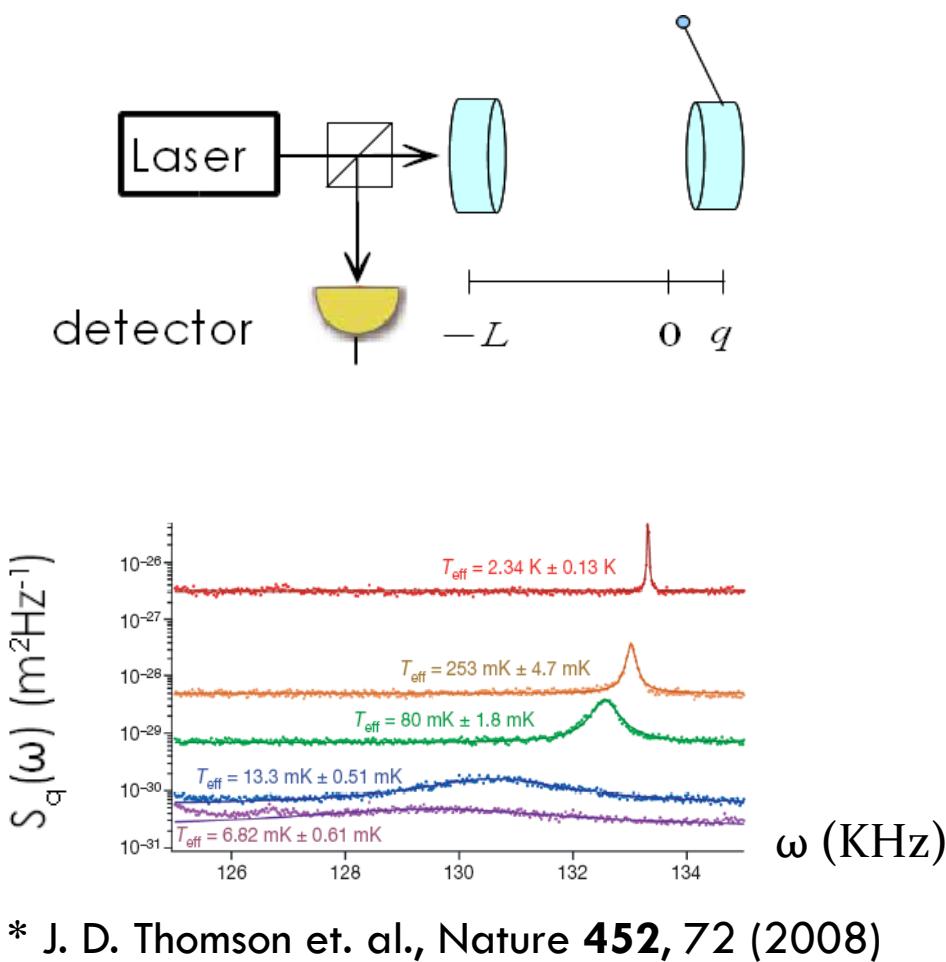
Resolved sideband limit : $\omega_m \geq \gamma$



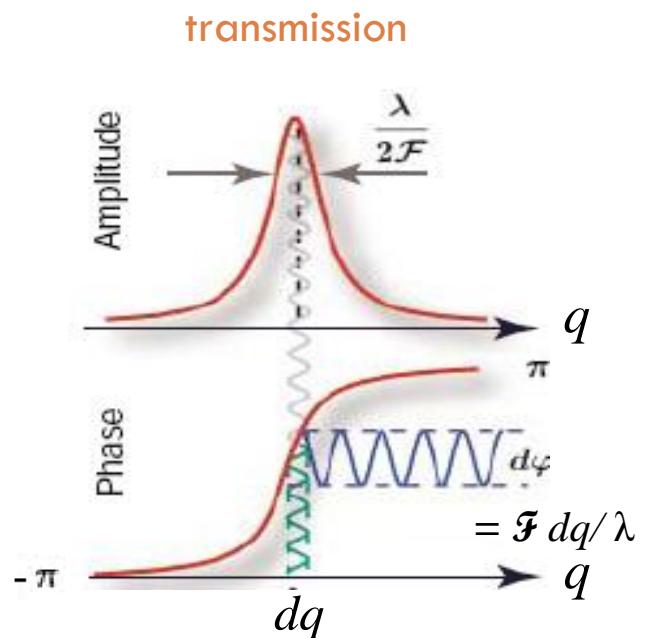
Theory : Marquardt, Girvin, PRL 2007

Experiment : Painter (2011), Kippenberg (2010)

SENSING OSCILLATOR MOTION



* J. D. Thomson et. al., Nature **452**, 72 (2008)

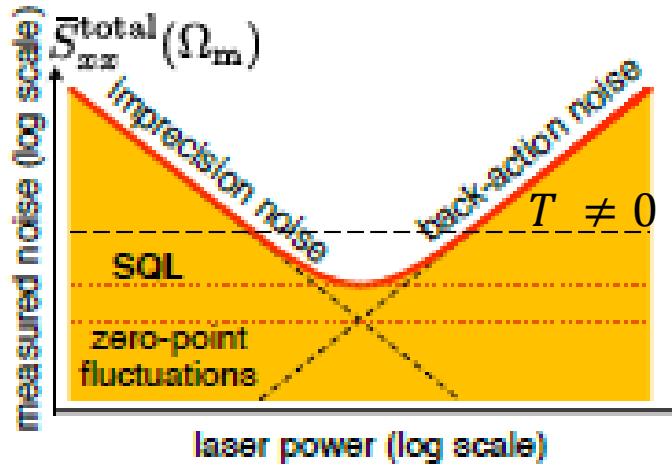


* Kippenberg, Vahala, Science (2008)

$$\int S_q(\omega) d\omega = \langle q^2 \rangle$$

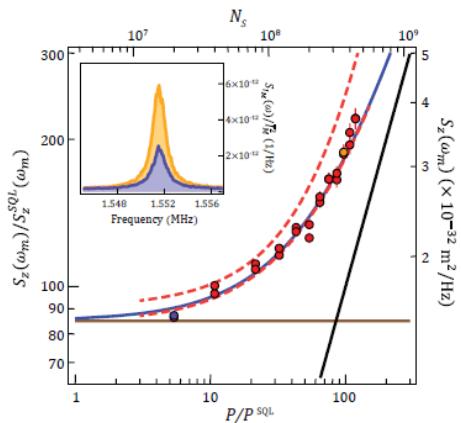
QUANTUM MECHANICS OF THE POSITION MEASUREMENT OF A MACROSCOPIC OBJECT

Optical phase fluctuations lead to excessive noise at **low** laser power



Amplitude fluctuations lead to excessive noise at **high** laser power

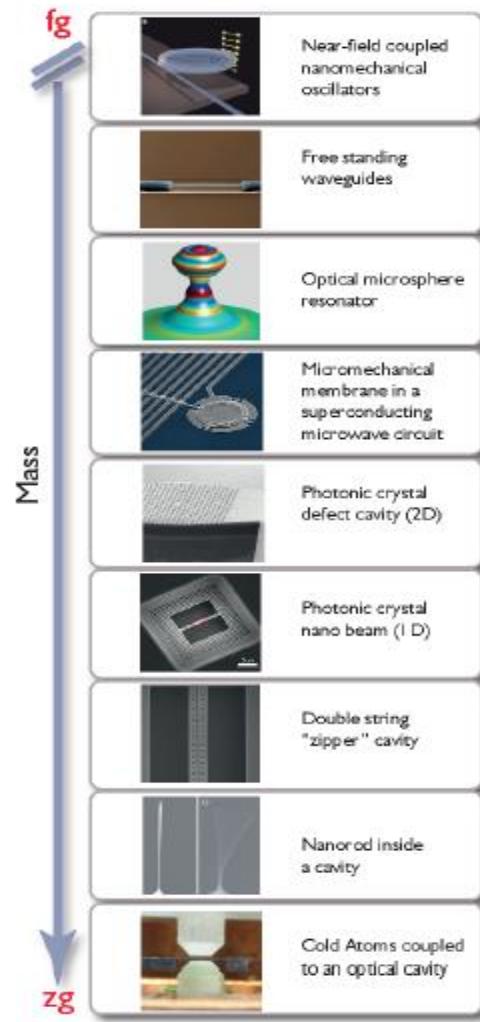
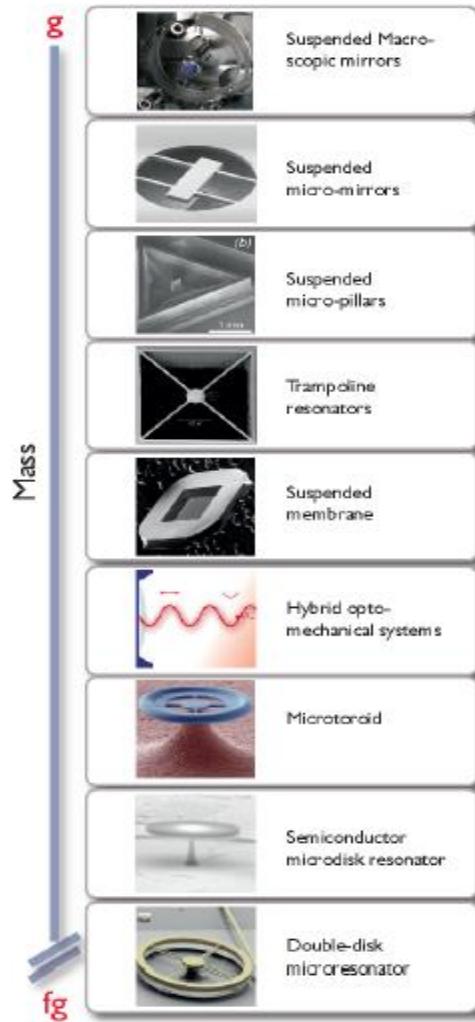
Observation of Radiation Pressure Shot Noise on a Macroscopic Object



- T. P. Purdy, R. W. Petersen and C. A. Regal,
Science **339**, 801 (2013).

Membrane: 0.5mm x 0.5 mm x 40nm, 7 ng
Temperature: 4.9K

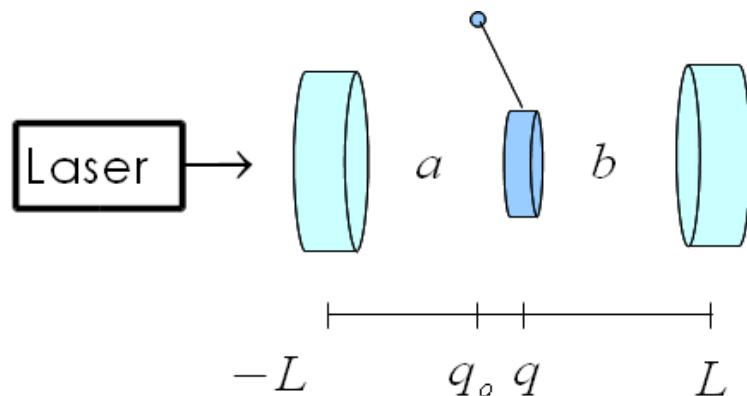
EXPERIMENTAL PLATFORMS



Different aspects of the light-matter interaction can be explored over many orders of magnitude in oscillator **mass** and **frequency**.

* Aspelmeyer, Kippenberg
and Marquardt:
[arXiv:1303.0733](https://arxiv.org/abs/1303.0733)

MEMBRANE IN A CAVITY



Earlier work :

- MB and P. Meystre,
PRL **99**, 073601 (2007)
- MB, H. Uys and P. Meystre,
PRA **77**, 033819(2008)

Recent:

- H. Shi and M. Bhattacharya,
PRA **107**, 043829 (2013)

Related experiments

1. Strong dispersive coupling of a high-finesse cavity to a micromechanical membrane

J. D. Thompson et. al

Nature **452**, 06715 (2008).

2. Optomechanically-induced transparency in a membrane-in-the-middle setup

M. Karuza, et. al,

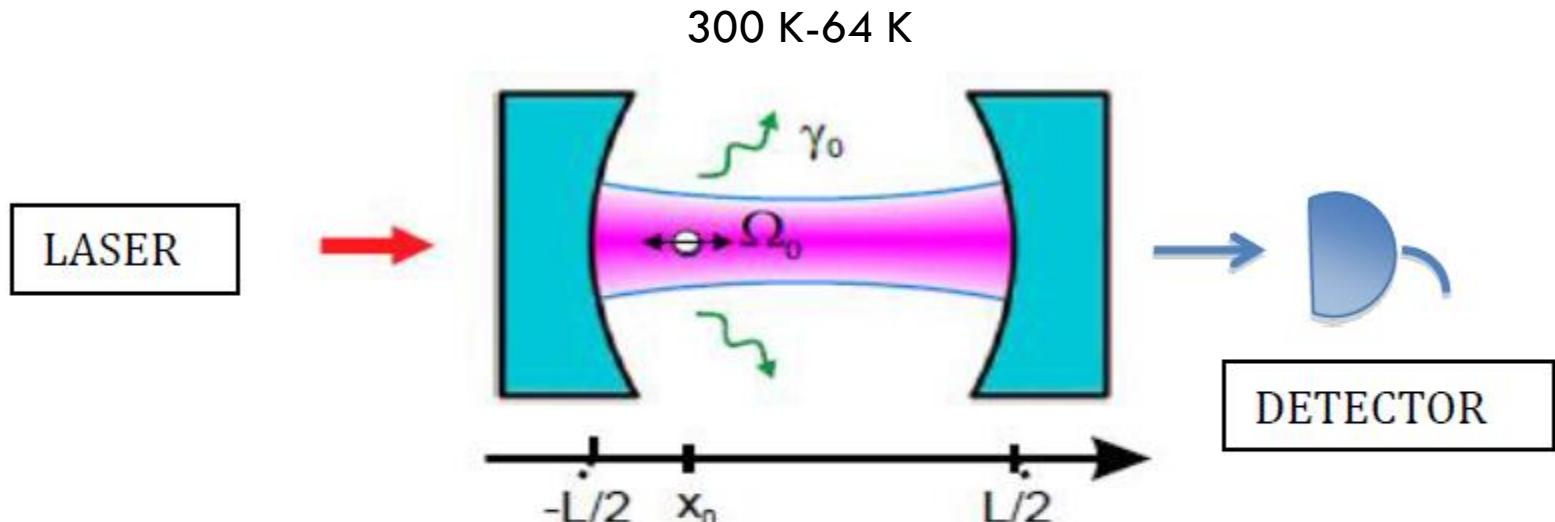
Phys. Rev. A **88**, 013804(2012)

Recent pedagogical article:

MB, H. Shi and S. Preble,
Am. J. Phys. **81**, 276 (2013).

DIELECTRIC IN A CAVITY

(a)



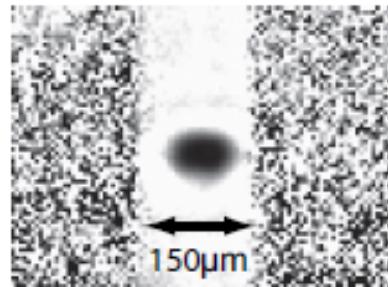
(b)



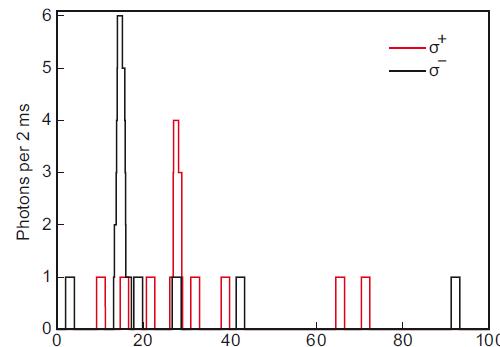
$$H \sim -d.E = -\alpha E^2/2 \longrightarrow \text{Optical trapping}$$

- * *Cavity cooling of an optically levitated submicron particle*
 - N. Kiesel et. al. PNAS USA, **110**, 14180 (2013)
 - Theory: A. Pflanzer et. al, PRA **86**, 013802 (2012)

ATOMIC BEC IN A CAVITY



BEC between the
mirrors after resonant
probing and 8 ms
time of flight



Periodic modulation of
an optical probe beam
transmitted by the cavity
due to **density modulations**
of the condensate

* *Cavity QED with a Bose-Einstein condensate*

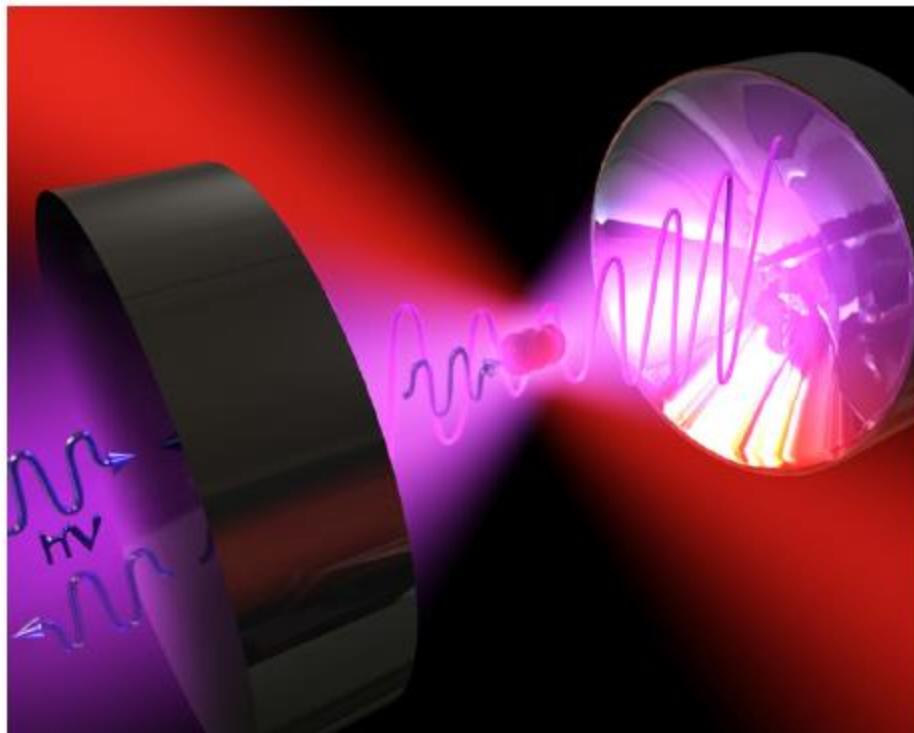
- F. Brennecke et. al, Nature **450**, 268 (2007)

Also: K. Zhang, W. Chen, MB, and P. Meystre, PRA **81**, 013802 (2010).

VIRUS IN A CAVITY: A PROPOSAL

Toward quantum superposition of living organisms

- O. Romero-Isart et. al, New Journal of Physics **12**, 033015 (2010)



Ashkin A and Dziedzic J M
*Optical trapping and manipulation
of viruses and bacteria*
Science **235**, 1517–20(1987)

Favorable living candidate:

Tobacco Mosaic Virus
50nm x 1 micron

TESTS OF QUANTUM MECHANICS

□ *Testing the superposition principle for massive objects*

VOLUME 91, NUMBER 13

PHYSICAL REVIEW LETTERS

week ending
26 SEPTEMBER 2003

Towards Quantum Superpositions of a Mirror

William Marshall,^{1,2} Christoph Simon,¹ Roger Penrose,^{3,4} and Dik Bouwmeester^{1,2}

□ *Testing the limits of the Schrodinger equation*

PRL 107, 020405 (2011)

PHYSICAL REVIEW LETTERS

week ending
8 JULY 2011



Large Quantum Superpositions and Interference of Massive Nanometer-Sized Objects

O. Romero-Isart,¹ A. C. Pflanzer,¹ F. Blaser,² R. Kaltenbaek,² N. Kiesel,² M. Aspelmeyer,² and J. I. Cirac¹

¹Max-Planck-Institut für Quantenoptik, Hans-Kopfermann-Strasse 1, D-85748, Garching, Germany

²Vienna Center for Quantum Science and Technology, Faculty of Physics, University of Vienna,

Boltzmanngasse 5, A-1090 Vienna, Austria

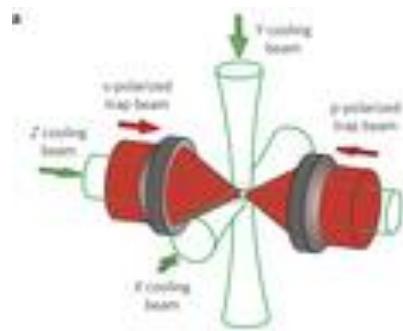
(Received 21 March 2011; published 7 July 2011)

CAVITYLESS COOLING

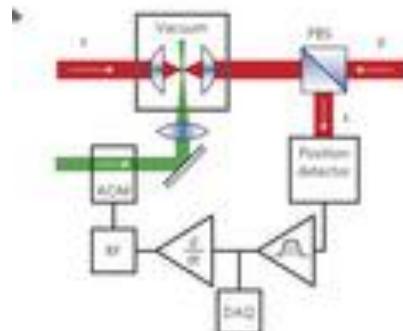
Millikelvin cooling of an optically trapped microsphere in vacuum

T. Li, S. Kheifets & M. G. Raizen

Nature 7, 527 (2011)



$$r = 2\mu\text{m}$$

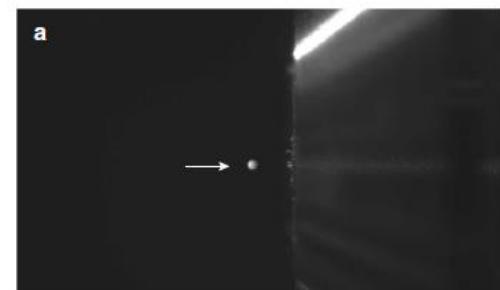


$$300\text{K} \rightarrow 1\text{ mK}$$

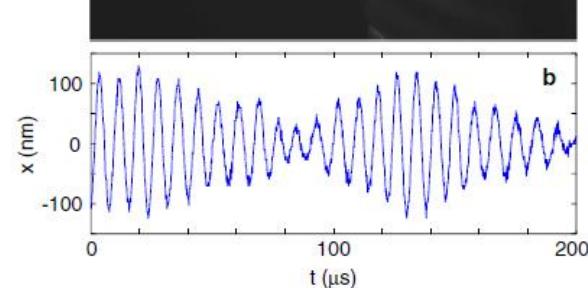
Subkelvin parametric feedback cooling of a laser trapped nanoparticle

J. Gieseler, B. Deutsch, R. Quidant & L. Novotny

PRL 109, 103603 (2012)

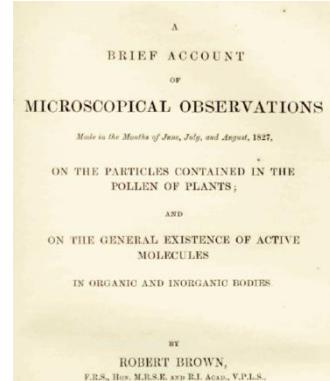
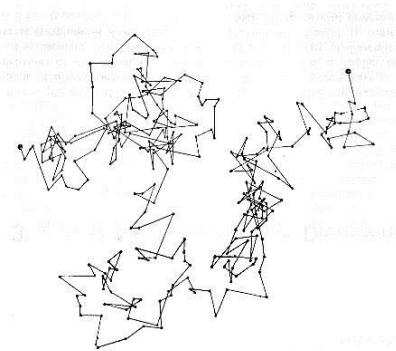
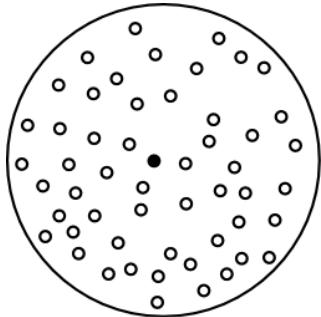


$$r = 70\text{nm}$$



$$300\text{K} \rightarrow 50\text{ mK}$$

BROWNIAN MOTION



Langevin (1908)

$$\tau_p = \frac{m}{\gamma}$$



Stokes viscous
drag coefficient

Ballistic regime: $(t \ll \tau_p)$: $\overline{x^2} = \left(\frac{k_B T}{m}\right) t^2$

Diffusive regime: $(t \gg \tau_p)$: $\overline{x^2} = \left(\frac{2k_B T}{\gamma}\right) t$

A. Einstein: *Annalen der Physik* **17** (8): 549(1905)

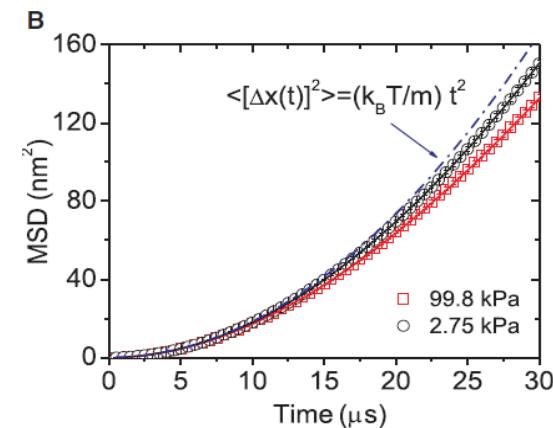
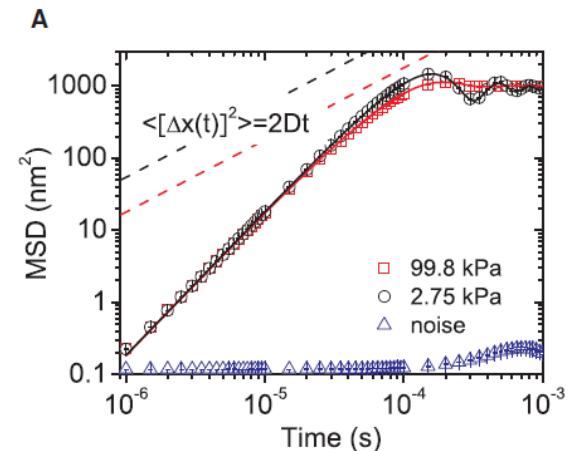
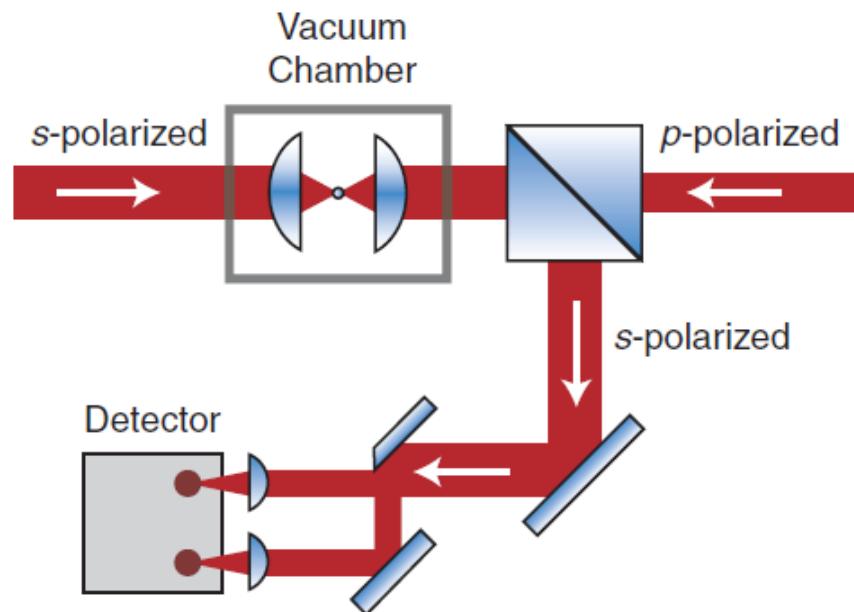
M. Smoluchowski: *Annalen der Physik* **21** (14): 756(1906)

Jean Perrin : Nobel Prize 1926

BROWNIAN MOTION

"Measurement of the Instantaneous Velocity of a Brownian Particle."

T. Li, S. Kheifets, D. Medellin, and M.G. Raizen. *Science*, 328, 1673 (2010)



CAVITYLESS COOLING

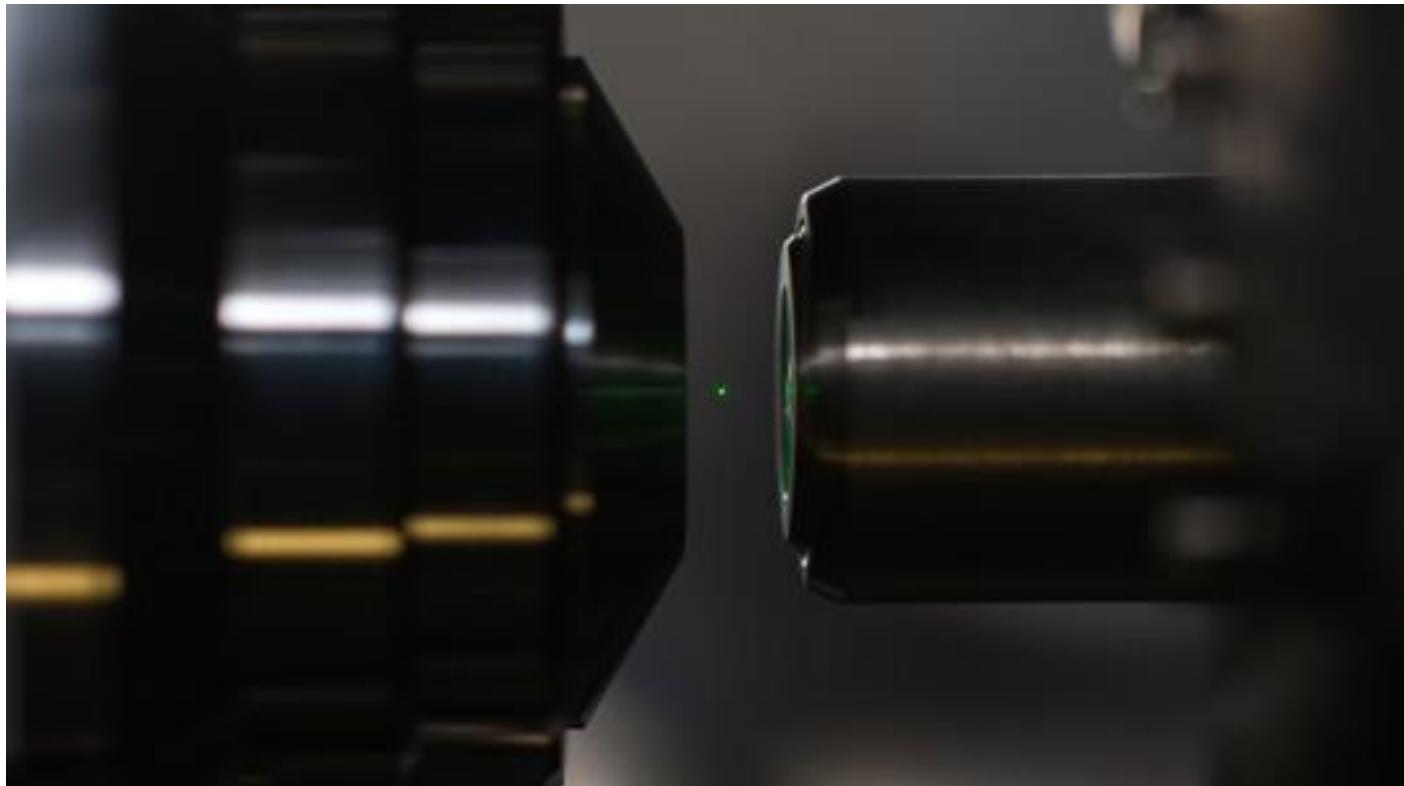
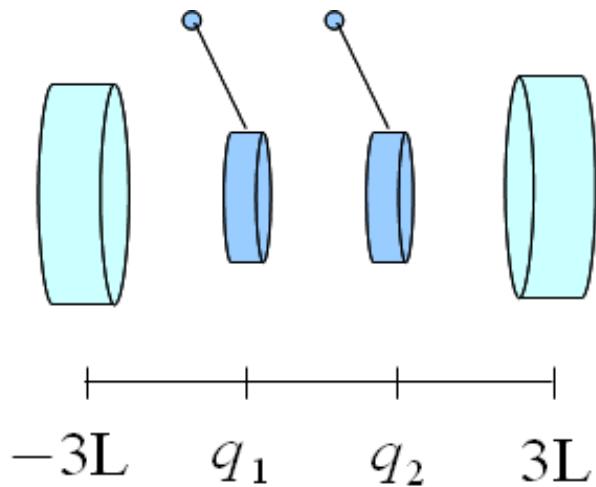


Image courtesy(Scientific American): [A. N. Vamivakas, University of Rochester](#)

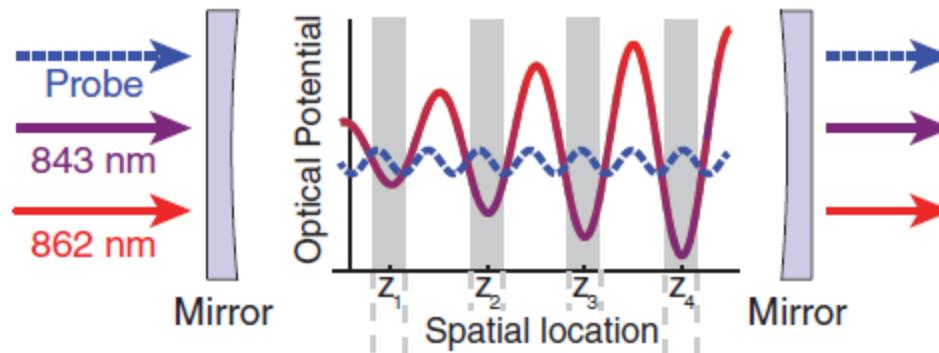
MULTIPLE MECHANICAL ELEMENTS



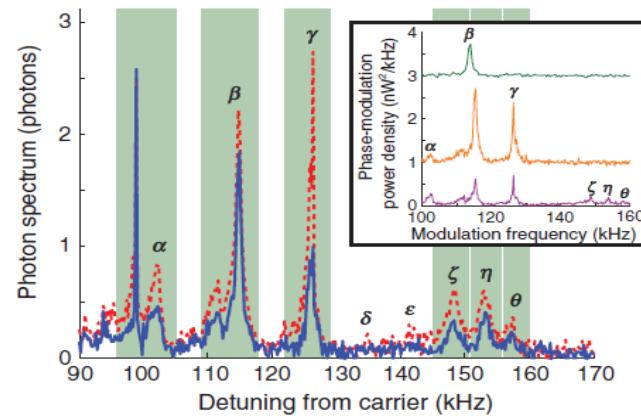
- MB and P. Meystre,
PRA **78**, 041801R (2008)
- M. Hartmann and M. Plenio,
PRL **101**, 200503 (2008).

Relative and center-of-mass motion can be detected
at different mechanical frequencies

MULTIPLE OSCILLATORS

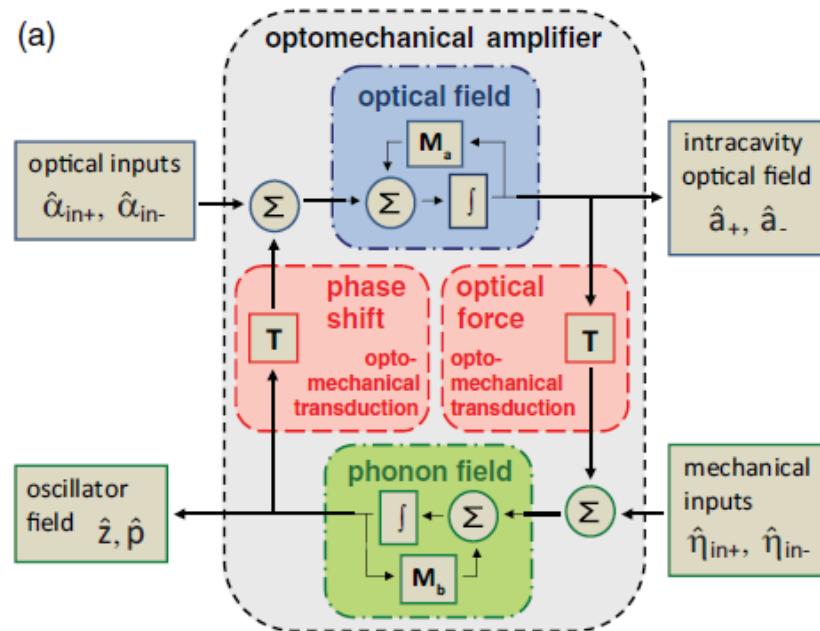


Occupation numbers
for 6 sites range
from 0.9-2.2



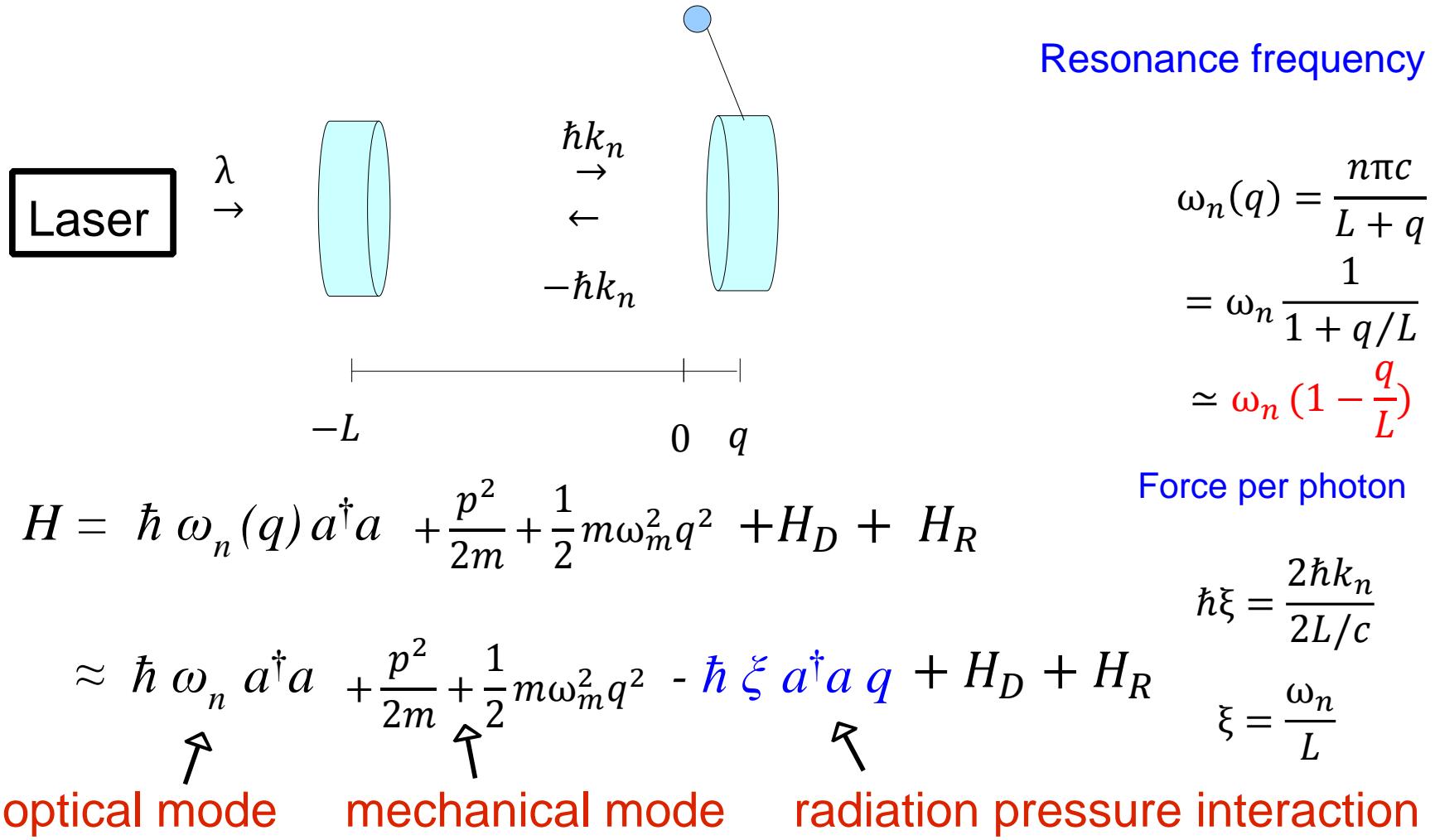
- * Optical readout of the quantum collective motion of an array of atomic ensembles
 - T. Botter et. al, PRL 110, 153001 (2013).

A TRANSDUCER FOR FLUCTUATIONS



- “*Linear amplifier model for optomechanical systems*”,
T. Botter et. al., PRA **85**, 013812 (2011)
- “*Coherent quantum-noise cancellation for optomechanical sensors*”,
M. Tsang and C. M. Caves, PRL **105**, 123601 (2010)

A TASTE OF THE THEORY



* C. K. Law, PRA **51**, 2537 (1995), H. Shi and MB, PRA **107**, 043829 (2013)

THE QUANTUM LANGEVIN EQUATIONS

$$H = \hbar\omega_c a^\dagger a + \frac{1}{2}\hbar\omega_m(p^2 + q^2) - \hbar G_0 a^\dagger a q + i\hbar E(a^\dagger e^{-i\omega_0 t} - a e^{i\omega_0 t})$$

Quantum Langevin Equations

$$\begin{aligned}\dot{q} &= \omega_m p, \\ \dot{p} &= -\omega_m q - \gamma_m p + G_0 a^\dagger a + \xi, \\ \dot{a} &= -(\kappa + i\Delta_0)a + iG_0 a q + E + \sqrt{2\kappa}a^{in},\end{aligned}$$

Noise correlators:

$$\langle \xi(t)\xi(t') \rangle = \frac{\gamma_m}{\omega_m} \int \frac{d\omega}{2\pi} e^{-i\omega(t-t')} \omega \left[\coth\left(\frac{\hbar\omega}{2k_B T}\right) + 1 \right]$$

$$\langle a^{in}(t)a^{in,\dagger}(t') \rangle = [N(\omega_c) + 1]\delta(t-t')$$

$$N(\omega_c) = (\exp\{\hbar\omega_c/k_B T\} - 1)^{-1}$$

→ The QLEs preserve the fluctuation-dissipation theorem

Generally non-Markovian

LINEAR RESPONSE

Semiclassical ansatz: Classical steady state + small quantum fluctuations

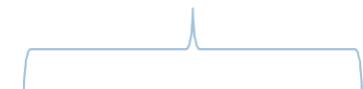
$$\delta\dot{q} = \omega_m \delta p,$$

$$\delta\dot{p} = -\omega_m \delta q - \gamma_m \delta p + G \delta X + \xi,$$

$$\delta\dot{X} = -\kappa \delta X + \Delta \delta Y + \sqrt{2\kappa} X^{in},$$

$$\delta\dot{Y} = -\kappa \delta Y - \Delta \delta X + G \delta q + \sqrt{2\kappa} Y^{in}$$

Field quadratures



$$\delta X \equiv (\delta a + \delta a^\dagger) / \sqrt{2}$$

$$\delta Y \equiv (\delta a - \delta a^\dagger) / i\sqrt{2},$$

* C. Genes et. al, PRA **77**, 033804 (2008)

Yields good agreement with experimental data for

1. Ground state cooling
2. Standard quantum limit
3. Squeezing of optical quadratures



$$U = \frac{\hbar\omega_m}{2} [\langle \delta q^2 \rangle + \langle \delta p^2 \rangle] \equiv \hbar\omega_m \left(n_{eff} + \frac{1}{2} \right)$$

Also yields covariance matrix for Gaussian states : useful for quantifying entanglement

MASTER EQUATION

Using the **Born** (weak coupling) and **Markov** (memory-less bath) approximations, a Lindblad-type master equation can be derived

$$\begin{aligned}\dot{\rho} = & -\frac{i}{\hbar}[\rho, H_1] + \frac{\gamma_o}{2}(2a\rho a^\dagger - a^\dagger a \rho - \rho a^\dagger a) \\ & + \frac{\gamma_m}{2}(\bar{n}_m + 1)(2b\rho b^\dagger - b^\dagger b \rho - \rho b^\dagger b) \\ & + \frac{\gamma_m}{2}\bar{n}_m(2b^\dagger \rho b - b b^\dagger \rho - \rho b b^\dagger),\end{aligned}$$

Advantage: Yields linear (coupled) equations for the elements of the density matrix
Disadvantage: Computationally hard to model classical-to-quantum transition as a large number of optical and mechanical states are involved.

NON-MARKOVIAN EFFECTS

Oscillator-bath interaction

$$H_{\text{int}} = q \sum_n c_n q_n$$

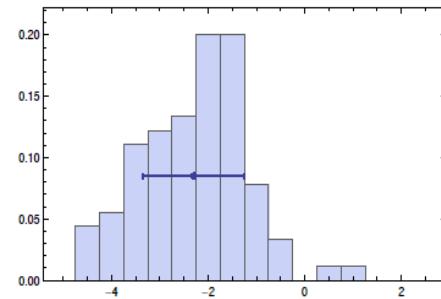
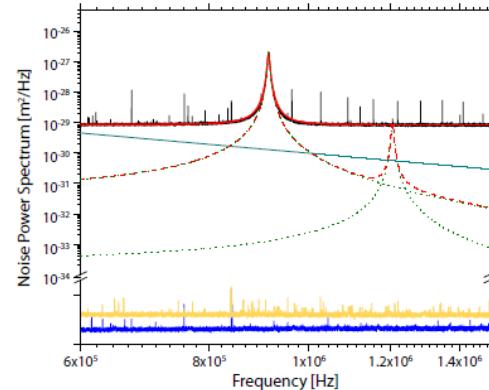
Noise spectrum

$$S_{\delta Y^{\text{out}}}(\omega) \approx c \frac{I(\omega)}{\omega ((\Omega_\infty^2 - \omega^2)^2 + (\gamma_\infty \omega)^2)}$$

Spectral density of thermal bath

$$I(\omega) = C \omega^k$$

$k = 1$: Ohmic
Markovian
dynamics



$$k \sim -2.3$$

Observation of non-Markovian micro-mechanical Brownian motion

S. Gröblacher,^{1,2} A. Trubarov,² N. Prigge,³ M. Aspelmeyer,^{2,3} and J. Eisert³

[arXiv:1305.6942](https://arxiv.org/abs/1305.6942) (2013)

WHAT WE LEFT OUT...

1. Nonlinear optomechanical coupling: proportional to $a^\dagger a q^2$ and higher powers of q .
2. Quantum non-demolition measurement of energy and motional quadratures.
3. Dissipative production of quantum states.
4. Information storage and retrieval.
5. Slow light and optomechanically-induced transparency.
6. Velocimeters, accelerometers, magnetometers, gyroscopes, thermometers...
7. Torsional and rotational degrees of mechanical freedom, which can be addressed using light's angular momentum.
8. Lots more.....

THANKS FOR YOUR ATTENTION !!

- The quantum-classical border is being redrawn.
- We are learning about “open” quantum systems.
- In the near future experiments should yield further insight and new technologies.
- For now, it seems that the laws of quantum mechanics hold in the macroscopic world...under the appropriate conditions.

~~~~~